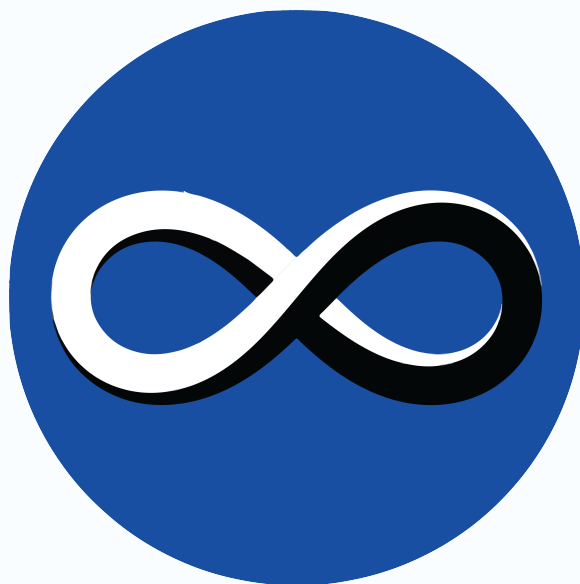


METH RECIPE BOOK

2026-27



MATHEMATICS CLUB



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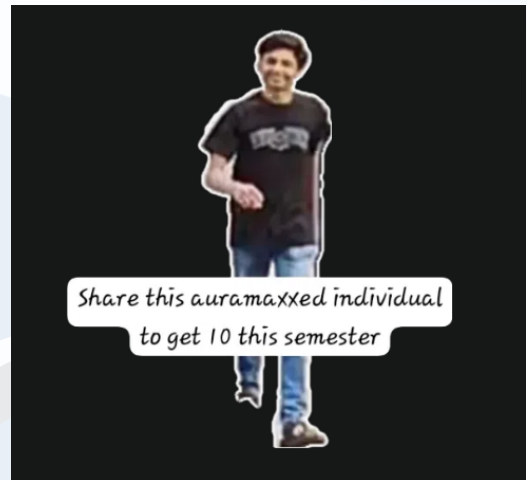
§0 Section-wise Chefs

If you have any doubts, please ask the corresponding coordinators:

- (a) Visual Learning - Aarav Prakash **+91 81518 42542**
- (b) A Good Time with Mathematics and Friends - Kailash Anand **+91 97909 08101**
- (c) Information Arithmetic - Harish M **+91 73059 78155**
- (d) JoJo and Juice - Vanka Sai Sriram Satvik **+91 97013 62223**
- (e) Geometry Meets Numerical Analysis - Ravi Karthik **+91 99630 01835**
- (f) A Spherical Odyssey - Prajith Saravanan **+91 89516 19547**



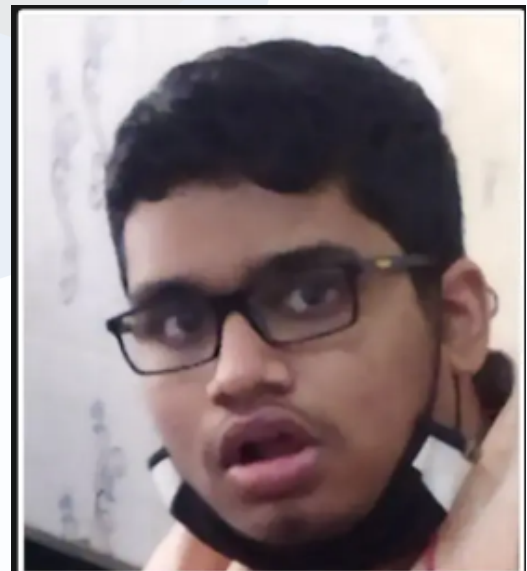
Pai Goat says all the very best!



Aura says hi!

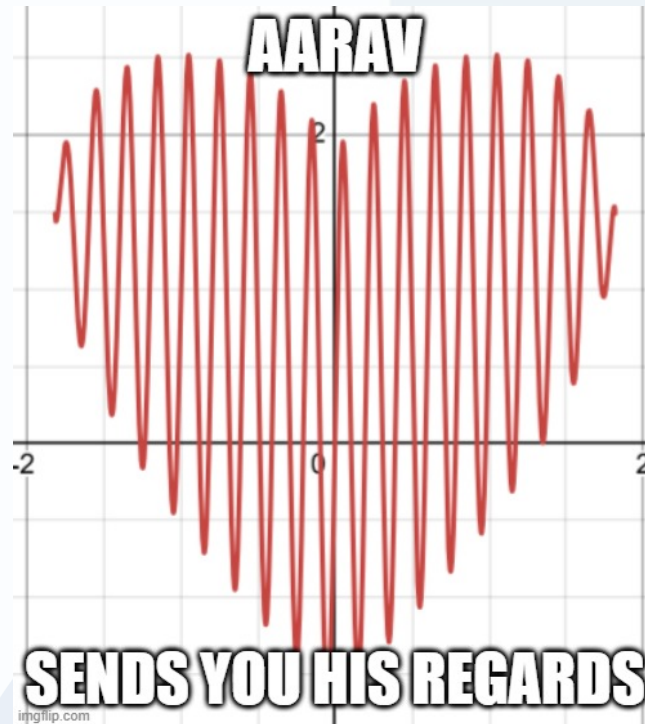


Eppa sends you best wishes



Use AI responsibly, or he'll belt u in interview.

§1 Visual Learning



Tools:

- (a) [GeoGebra Classic](#) (best for points sliders and the trace tool)
- (b) [Desmos](#) (best for quick equations and parametric curves). Both are free and run in the browser.

Rotation Matrices: Spinning the Plane

A matrix is a machine that eats a point and spits out a new one. The remarkable fact is that a *linear* machine is completely determined by what it does to just two arrows, $(1, 0)$ and $(0, 1)$: every other point is a recipe of “this much of the first arrow plus that much of the second”, and the machine has to respect the recipe.

Explore:

- (a) 3Blue1Brown - [Linear transformations and matrices](#) (the best 10 minutes you will spend on linear algebra)
- (b) Wikipedia - [Rotation matrix](#)

The Sliding Ladder: Loci

A *locus* is the path a point traces when it is forced to obey a rule. Loci are where geometry and algebra meet: the rule is geometric, the path has an equation, and proving they match is the whole game.

Explore (after you have tried it).

- (a) This exact device has a name and a long history as a drawing tool: look up the [Trammel of Archimedes](#).

The crankshaft: turning circles into lines

A car engine has a problem: the fuel explodes in a straight line (pushing a piston), but the wheels need to spin. The *slider-crank* linkage solves it, converting back-and-forth motion into rotation and back. It is so natural that you might expect the piston to move in a perfect sine wave. Does it?

Explore more:

- (a) Wikipedia, [Slider-crank linkage](#) (with animations).

Bézier curves: the shape of every font

In the 1960s, two engineers at rival French car makers, Pierre Bézier at Renault and Paul de Casteljau at Citroën, needed a way to describe smooth car bodies to computers. Their curves now draw every letter on your screen, every path in Illustrator and Figma, and most animations on the web. A cubic Bézier curve is controlled by just four points, and two of them are never even on the curve.

More Resources:

- (a) Freya Holmér, [The Beauty of Bézier Curves](#) (a stunning visual introduction)
- (b) Pomax, [A Primer on Bézier Curves](#) (everything, with interactive figures)
- (c) [Interactive Bézier spline tool](#) (for the bonus).

§2 A Good Time with Mathematics and Friends

Me and the homies discussing real analysis
wondering how $1 + 1 = 2$



My Thoughts on Real Analysis:

There is a chance that you might think real analysis is extremely boring and pointless. I mean, of course $0 \neq 1$, and of course $1 + 1 = 2$. What's the point of proving this?

I think real analysis offers a new perspective. We use numbers like 0, 1 all the time in our lives, and it has a physical meaning for us. 1 of something means that the quantity of that something is singular. Similarly, 1.5 hours means it's been an hour and thirty minutes. We have such an intuitive feeling for numbers; it just runs in our blood. While doing any abstract algebra work, it is only natural that this intuitive feeling we have for numbers carries over. And that's good! That's exactly what motivated all these axioms and definitions in the

first place - intuition! Think about this very deeply, mathematically - what even are numbers? I think about them as mathematical abstractions of the human intuition to measure stuff. So if you were creating real analysis for the first time in human history, you'd want the axioms you define to reflect our intuition for numbers, while maintaining a distance to their physical manifestations.

So in the world of math, $0_{\mathbb{F}}$ is not "the absence of stuff". $0_{\mathbb{F}}$ is the element in $\mathbb{F}$ that when added to any $x \in \mathbb{F}$ gives back x itself. It is the *additive identity*. And $-x$ is not a "negative number" it is a symbol for the element $y \in \mathbb{F}$ that when added to x gives $x + y = 0_{\mathbb{F}}$ - it is the *additive inverse*. So this naturally leads us to the question - "Are we sure that there exists only a single $y \in \mathbb{F}$ that does this? Is there not another $z \in \mathbb{F}, z \neq y$ that does this?"

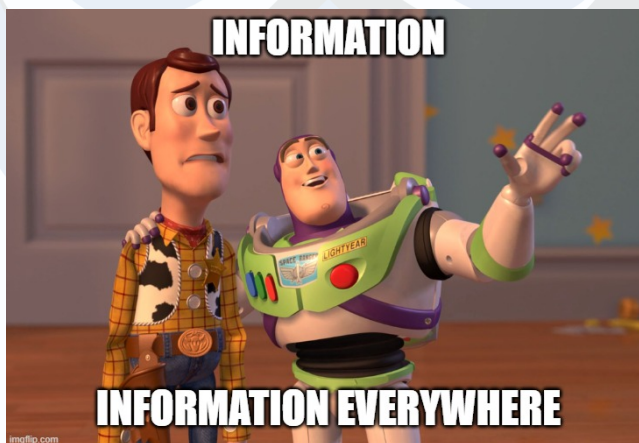
The answer seems obvious - "Of course there is only a single $-\pi$. Adding anything else to π won't give 0", you're probably saying. Stop thinking in terms of the numbers you know! That's why I'm sub-scripting every 0 with $0_{\mathbb{F}}$ and 1 with $1_{\mathbb{F}}$ - so that you remember that these are just elements in a set that have some property.

So when I ask: "Is $-1_{\mathbb{F}} \cdot x = -x$ ", I'm not asking is $-1 \cdot 2 = -2$? What I'm asking is - " $1_{\mathbb{F}}$ is the multiplicative identity that gives back x when they are multiplied together. Does the additive inverse of the identity (denoted by the symbol $-1_{\mathbb{F}}$) give the additive inverse of x when multiplied together?"

Textbook Resources

- (a) A nice introduction to real analysis: [Elementary Real Analysis by Brian S. Thomson, Judith B. Bruckner, and Andrew M. Bruckner](#)
- (b) The ideas behind mathematical proof writing: [Book of Proof, Richard H. Hammack](#)

§3 Information Arithmetic



Since we will be dealing with some probability here, here are a few terms that will be useful.

- A **random variable** X assigns a real number to each outcome of an experiment. For example, if you roll two dice, X could be their sum. It is *discrete* if it takes separate values (like $2, 3, \dots, 12$) and *continuous* if it can take any value in an interval (like a height, or a time).

- For a discrete X , the **probability mass function** gives the probability of each value directly: $p_X(x) = P(X = x)$.
- For a continuous X , any single exact value has probability zero, so we use a **probability density function (PDF)** $f_X(x)$ instead. It measures how densely probability is packed near x :

$$P(x \leq X \leq x + \Delta x) \approx f_X(x) \Delta x \quad \text{for small } \Delta x.$$

A density is not a probability, and it can be larger than 1. It only needs $f_X(x) \geq 0$ and $\int_{-\infty}^{\infty} f_X(x) dx = 1$.

- Probabilities come from integrating the density. The **cumulative distribution function (CDF)** is

$$F_X(x) = P(X \leq x) = \int_{-\infty}^x f_X(t) dt,$$

so $F'_X(x) = f_X(x)$, and for any range

$$P(a \leq X \leq b) = \int_a^b f_X(t) dt = F_X(b) - F_X(a).$$

- **Example.** If $f_X(x) = \frac{e^x}{e-1}$ for $0 < x < 1$ (and 0 elsewhere), then $F_X(x) = \frac{e^x - 1}{e - 1}$ on that interval, so $P(X \leq 0.7) = \frac{e^{0.7} - 1}{e - 1} \approx 0.59$, while $P(X = 0.7) = 0$.

For more info, you can refer to the following references:

- [Probability | A Simple Introduction for Beginners](#)
- [Probability Course | probabilitycourse.com | H. Pishro-Nik](#)

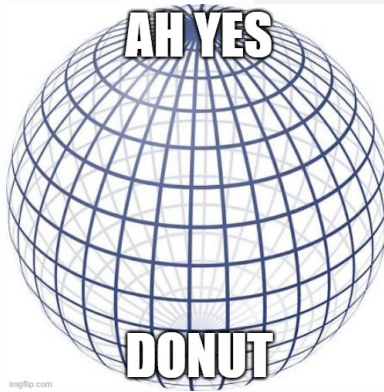
For the main section, here are some resources for your reference:

- [3Blue1Brown - Solving Wordle using Information Theory](#)
- [Khan Academy - Information theory | Computer science theory](#)
- [Art of the Problem — "Information Theory: The Language of Coins"](#)
- Computerphile's "Huffman Encoding" and "Shannon Limit" videos.

Also, do check out:

- Huffman Coding - [Huffman Coding Algorithm](#)
- Joint and Conditional Entropy
- KL Divergence and Cross-Entropy and their usage in LLMs

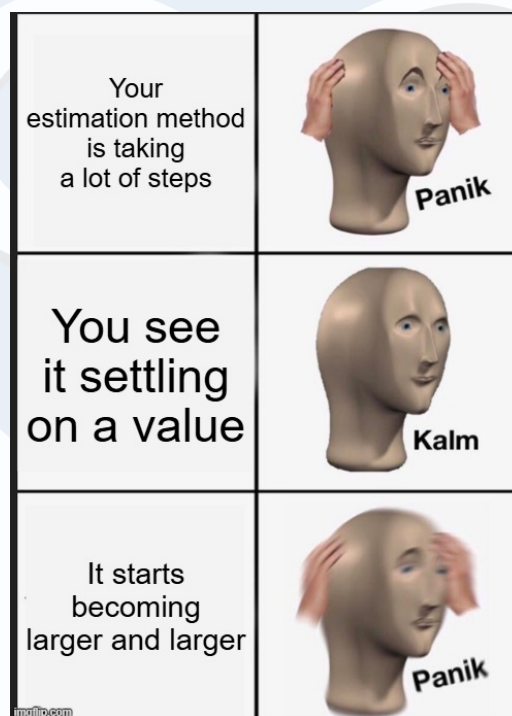
§4 JoJo & Juice



Don't say you love the anime, if you haven't read the manga

- Refer to the links given below to help yourself understand the question better.
- [Math Club IITM | DC Application Hints | Satvik](#)
- Some other ~~historical~~ hints. [History of Graphs](#)

§5 Geometry Meets Numerical Analysis



So, you feel like estimating things eh? Well, there's a lot of ways you could go about it (which you will learn in your upcoming semesters, so don't take this lite).

But first, a small refresher. Ever heard of the **harmonic mean**? If not, let me tell you about it.

The harmonic mean of two elements is said to be the *inverse of their inverses' arithmetic mean*. Still confused? A formula should help you figure it out:

$$H_{a,b} = \frac{2}{\frac{1}{a} + \frac{1}{b}} = \frac{2ab}{a+b}$$

This has an interesting property when you graph it around the root. What do you think it could be? ~~What happens to the signs?~~

Another mean that is useful is the **Root Mean Squared** (RMS). This is basically what it says, the root of the mean of the squares:

$$RMS_{a,b} = \sqrt{\frac{a^2 + b^2}{2}}$$

So, segueing into estimation methods, how do we do this? Well, there are many ways, but we will stick to a few of them right now:

Fixed-Point Iteration: Basically, the easiest to know and implement. This is an iterative method where given an input x_0 , you modify the original equation $f(x) = 0$ in a way that makes it into $g(x) = x$, and by repeatedly applying $g(x)$ to x_0 , you get a point where $|g(x_i) - x_i| \leq \epsilon$, and you've found your root!

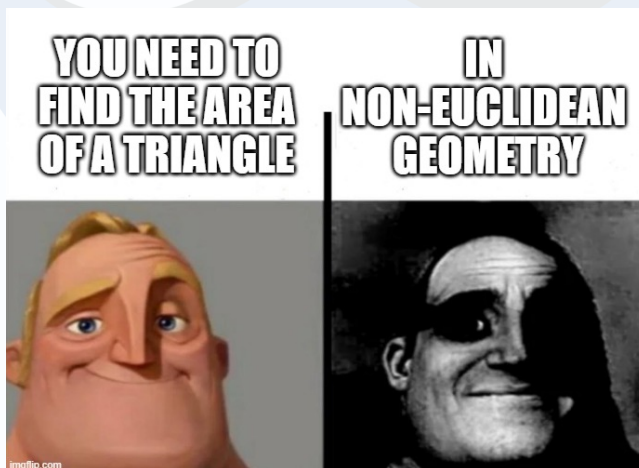
Newton-Raphson: This is a very interesting estimation method. Using derivatives, you can approximate $y = f(x)$ by its tangent, and that gives you the next approximation. The formula for the next iteration is:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Rinse and repeat until you have enough precision.

Note: The error here becomes smaller quadratically (Why?) as the iterations go on, often being referred to as a **quadratic convergence**.

§6 A Spherical Odyssey



Conventions

- The Earth is a sphere of radius $R = 6371$ km. Latitudes are N, longitudes are E.

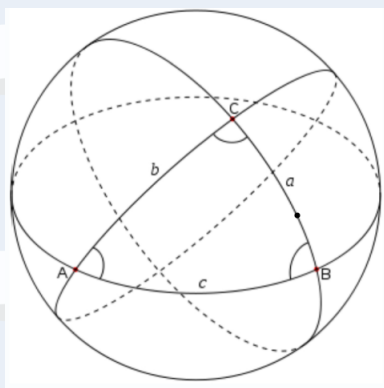
- Bearings are measured clockwise from north ($000^\circ = \text{N}$, $090^\circ = \text{E}$, $180^\circ = \text{S}$, $270^\circ = \text{W}$).
- **Heading and ground track.** The *heading* is the direction the ship points. The *ground track* is the path it really follows over the Earth. Between two stops the crew keeps the same heading, unless the text says otherwise. Treat every ground track as a great-circle arc. A bearing given for a track is its bearing at the start.
- **Wind drift.** A wind imparts a *drift velocity* $\mathbf{u}$ to anything afloat. A ship's velocity over the ground is

$$\mathbf{v}_{\text{ground}} = \mathbf{v}_{\text{ship}} + \mathbf{u},$$

where $\mathbf{v}_{\text{ship}}$ is its velocity through the water, of constant magnitude $v_s = 8 \text{ km/h}$ and pointing wherever the crew steers. A raft with no sail and no oars has $\mathbf{v}_{\text{ship}} = \mathbf{0}$, so it moves with *pure wind drift*, $\mathbf{v}_{\text{raft}} = \mathbf{u}$.

- **Poseidon's curse:** from the moment Prajithsseus leaves Troy, a constant westerly wind blows. At every point on Earth it blows toward *local east* (drift direction 090° at that point), with a fixed speed u that is unknown. It acts on everything afloat *except* while Aeolus's bag is sealed (see Part II).
- **Unsteered drift.** An object that nobody steers is carried only by the wind, and since the wind always points toward local east, its path is a circle of latitude (a parallel), *not* a great circle.

Useful formulae (spherical trigonometry)



The Spherical Triangle

Consider a spherical triangle on the unit sphere with sides a, b, c (each side is the angle it subtends at the centre of the sphere, so an arc of length s on our Earth has $a = s/R$) and opposite angles A, B, C . Then

$$\text{Law of cosines for sides: } \cos a = \cos b \cos c + \sin b \sin c \cos A,$$

$$\text{Law of sines: } \frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

Note: A velocity of speed v and bearing θ has components $(v \sin \theta, v \cos \theta)$ in the (east, north) directions. Velocities add as vectors. Conversely, a velocity with components (g_E, g_N) has bearing $\text{atan2}(g_E, g_N)$ (add 360° if the result is negative). Here $\text{atan2}(y, x)$ is the two-argument arctangent: the angle, in $(-180^\circ, 180^\circ]$, of the point (x, y) measured from the positive x -axis. Unlike $\arctan(y/x)$ it uses the signs of both x and y , so it always lands in the correct quadrant.

Deriving the navigation formulae:

Let N be the North Pole, and let $P_1 = (\varphi_1, \lambda_1)$ and $P_2 = (\varphi_2, \lambda_2)$ be two points on the Earth, with $\Delta\lambda = \lambda_2 - \lambda_1 \in (-180^\circ, 180^\circ]$. Let $\delta = s/R$ be the angular length of the great-circle arc P_1P_2 , and let θ be its initial bearing at P_1 . Draw the spherical triangle NP_1P_2 and use *only* the two laws above.

- (a) *Distance and bearing.* Given P_1 and P_2 , find δ and θ in terms of $\varphi_1, \varphi_2, \Delta\lambda$. Explain how to decide whether the bearing is θ or $360^\circ - \theta$.
- (b) *Destination point.* Given P_1 , the bearing θ and the arc length s , find φ_2 and $\Delta\lambda$. Explain how to avoid the quadrant ambiguity of arcsin when finding $\Delta\lambda$.

These two results are all the spherical trigonometry you need for the rest of the problems. You don't have to submit the addendum questions.

References and further reading

- (a) Homer, *The Odyssey*
- (b) W. M. Smart, *Textbook on Spherical Astronomy*, 6th edition, revised by R. M. Green (Cambridge University Press, 1977). See the opening chapter on spherical trigonometry.
- (c) I. Todhunter, *Spherical Trigonometry*, revised by J. G. Leathem (Macmillan, 1901).
- (d) [Spherical trigonometry | Wikipedia](#)
- (e) [Wolfram - Spherical Trigonometry](#).