

DEPUTY COORDINATOR APPLICATION

2026-27



MATHEMATICS CLUB



CENTRE FOR INNOVATION
IIT MADRAS

Instructions

For your reference, we have included the [addendum](#) here.

§1 General

- For all the questions you answer, explain your thought process and provide descriptive answers. Partial marks will be awarded for partial solutions.
- The internet, AI, and all other resources can be used to help you solve the application. Using Gemini, ChatGPT, and Claude can help you learn greatly; however, it is you who will give the interview.
- We do not expect you to fully solve the application. Do what you can.
- Collaboration is not permitted. Please cite your reference(s).
- The application is intended for you to enjoy learning the topics that we have included. Some resources for the app have been included in the [addendum](#) for your reference. All the best, and have fun!

§2 Application Format

- Solutions can be written in any medium that allows math, i.e., digital notes, documents, or $\text{\LaTeX}$. Upload all of your work, answers and images into a single PDF file and submit before the deadline.
- The mode of the submission will not be taken into account for your evaluation. So don't spend too much time worrying about beauty, and attempt the general section and as many sections as you like; we value depth over coverage.
- The answers are expected to be brief and elegant. Do not use filler content; explain concisely.
- The format for the PDF name is `<firstname>_<rollnumber(caps)>_MCApp.pdf`
(For example: Arjunachalam_MC26B067_MCApp.pdf).

§3 Submission

- The final PDF is to be submitted in this [Google Form](#).
- The deadline for submission is **25th October 2026, 11.59 pm**.
- Join this [WhatsApp group](#) for further notifications and updates.

§4 Contact Us

For any doubts, we have attached our heads' contacts here, and all section chefs' contacts can be found in the [addendum](#).

- Arjun Arunachalam: [+91 91507 16759](#)
- Laxmi Narasimha Reddy: [+91 81858 68798](#)

§0 General Questionnaire

- (a) Introduce yourself. What are your interests, hobbies, and your interesting traits. (*under 100 words*)
- (b) Why did you choose the Mathematics Club? What interests you about the club? (*under 100 words*)
- (c) What is your expectation for your DC tenure? (*under 100 words*)
- (d) Why do you like math? What topic do you like the most and which one do you hate the most in math? Why do they interest you and scare you, respectively? (*under 200 words*)
- (e) If you could propose your own mini-project for the club, which topics or domains would you like to explore, and why? (*under 100 words*)



Green Aarav (for no reason).

§1 Visual Learning

Refer to the [addendum](#) for resources.

Aarav, a huge computer nerd, swears by Desmos and GeoGebra being able to solve ANY problem (and he will die on this hill). Being the zealot that he is, he has now forced you to explore the wonders of graphs! In order to appease him, you have to solve these tasks.

All of these problems can be solved with GeoGebra or Desmos. GeoGebra is more complex, but also more powerful. You may use either tool for any question. Submit your graphs by sharing the links and also paste a screenshot of each one in your PDF along with the explanations (tell us why it is working).

The math you will use in this section is fundamental to the working of nearly all the technology you interact with everyday - from pixels on the screen to the pistons in your (or parents) car. As you go through this section, try to connect the dots between each question rather than looking at them as standalone problems. The central idea here is to describe a geometric problem using an equation and seeing how it behaves.

Q 1.1. What is a 2D rotation matrix? Where does it send the points $(1, 0)$ and $(0, 1)$, and why is that enough to write down the whole matrix?

- (a) Use the concept to rotate any graph by an arbitrary angle θ (you should put θ on a slider). Try it on $y = x^2$ and any other nice pattern of your choice.
- (b) Multiply $R(\alpha)$ and $R(\beta)$. What should you get? Notice anything familiar?
- (c) What does a rotation matrix look like in 3D? In 2D we rotate about a point and in 3D we rotate about an axis, so what do we rotate about in 4D? How many different "planes of rotation" are there in n dimensions?

Q 1.2. Create a line segment of length l such that one end slides along the positive x-axis and the other along the positive y-axis (think of a ladder sliding down a wall).

- (a) Find the locus of its midpoint, and prove it. *Hint: GeoGebra's trace tool can be very powerful here.*
- (b) What if you trace some other point on the ladder instead of the midpoint? Can you find the equation of the curves you get?

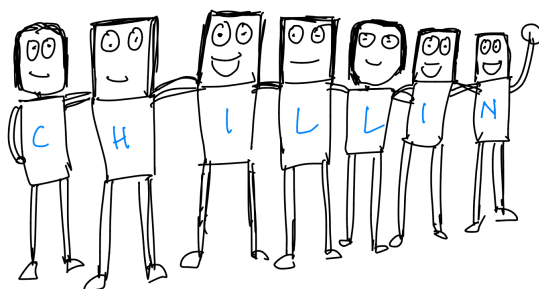
Q 1.3. Create a line segment of length l such that one end is locked to a point going around a circle of radius r centred at the origin, and the other end slides along the x-axis. Take $l > r$. (You might know this as the crank-shaft mechanism in a car engine!)

- (a) Find the x -coordinate of the sliding end as a function of the angle of rotation θ , and plot it against θ .
- (b) Does it produce a perfect sine wave? If not, why, and where is it the most off? What happens as the rod gets longer? And what goes wrong if $l < r$?
- (c) How can we make it produce a perfect sine wave?

Q 1.4. What is a Bézier curve? What do the 4 points used to define one do? Using 4 arbitrary points in GeoGebra, can you make a Bézier curve between them? *Hint: Bézier curves have many forms; some may be easier to implement than others!*

Bonus: Try stitching multiple Bézier curves end-to-end to form a Bézier spline. What do various alignments of the control points do in terms of the graph?

§2 A Good Time with Mathematics and Friends



After joining the Mathematics Club, the coordinators wanted to dive deep into the gears and cogs that make math what it is today. On a Friday night after dinner, they sat together in the ESB fan area and started questioning.

Kailash was in a philosophical mood that day. He asked, "What are numbers? I am not asking what motivated the creation of numbers - that probably would be counting cattle, ensuring a fair barter took place and so on. I'm asking a more fundamental question - what really *are* numbers?"

Prajith, having studied some real analysis before, shared his viewpoint: "In an abstract sense, numbers are elements of a set, such that all elements of that set satisfy some properties. In a way, numbers are *defined* by those said properties, called *the field axioms*."

Q 2.1. What are the field axioms? Give some examples of fields - both finite and infinite. Feel free to think up your own crazy field as well!

Q 2.2. (Bonus) Satvik noticed that the definition of multiplication here has nothing to do with being repeated addition! He asked everyone - "Guys, say you were a kindergarten teacher. How would you teach kids multiplication for the first time? Do you think doing it as repeated addition is the best way? Why or why not?"

Kailash wanted to take these axioms for a spin! "Let's see if these 'axioms' can really make a random 'set of elements' behave like the numbers are used to!" he said, excitedly.

Q 2.3. We know that 'you can move the minus sign around' while doing multiplication: $a \cdot (-b) = (-a) \cdot b$ where $a, b \in \mathbb{F}$. Can you prove this for an arbitrary field using only the field axioms?

- Q 2.4.** We know that $0 \cdot x = 0, \forall x \in \mathbb{R}$. In a general field, $0_{\mathbb{F}}$ is the additive identity. Do these axioms force $0_{\mathbb{F}} \cdot x = 0_{\mathbb{F}} \forall x \in \mathbb{F}$?
- Q 2.5.** The axioms exclude $0_{\mathbb{F}}$ while talking about multiplicative inverses, why? Can you show that $0_{\mathbb{F}}$ cannot have a multiplicative inverse? That is, Show $\nexists x \in \mathbb{F}$ such that $0_{\mathbb{F}} \cdot x = 1_{\mathbb{F}}$.

"Wait a second!", Harish realized. "How do we know that $0_{\mathbb{F}} \neq 1_{\mathbb{F}}$? If $0_{\mathbb{F}} = 1_{\mathbb{F}}$, then $0_{\mathbb{F}}$ *does* have a multiplicative inverse! In the special case $\mathbb{F} = \mathbb{R}$, we know $0_{\mathbb{R}} \neq 1_{\mathbb{R}}$ is true."

- Q 2.6.** What about in general, can $0_{\mathbb{F}} = 1_{\mathbb{F}}$? What happens to the elements in our field if we force this condition?
- Q 2.7.** Consider a field $\mathbb{F}$ where $0_{\mathbb{F}} \neq 1_{\mathbb{F}}$. What is the smallest possible cardinality of the set $\mathbb{F}$? Explicitly write out the addition and multiplication mappings for $\mathbb{F}$. Do these operations seem familiar? This field is the famous Galois Field of size 2: $\text{GF}(2)$!
- Q 2.8.** Let $\mathcal{M}$ be the collection of invertible $n \times n$ matrices over $\text{GF}(2)$ (This just means all the entries of the matrix are from $\text{GF}(2)$). Let $c_0 : \mathcal{M} \rightarrow \{0, 1, 2, \dots, n^2\}$ be the function that counts the number of zeros in a matrix M . Find $\min_{M \in \mathcal{M}} c_0(M)$, and $\max_{M \in \mathcal{M}} c_0(M)$. Are all matrices in $\mathcal{M}$ also invertible over $\mathbb{R}$?
- (Hint: Start small, and slowly get bigger)

§3 Information Arithmetic

Q 3.1. Bit by Bit

Anirudh and Prajith are playing a game. Anirudh thinks of a whole number from 1 to 1000. Prajith may only ask yes/no questions, and wants to pin the number down as fast as possible.

- (a) What is the fewest number of questions that *always* works? What does each answer do to the set of possibilities?
- (b) Satvik claims that an event of probability P carries

$$I = -\log_2 P \text{ bits}$$

of information. Why should a rare event tell you more than a common one? And why a *logarithm*, of all things?

Hint: Two independent events both happen. What should their information do?

- (c) Anirudh gets lazy: he picks 1 half the time, 2 a quarter of the time, and 3 or 4 an eighth of the time each. Can Prajith average fewer than 2 questions? Now average Satvik's I over all four outcomes. Coincidence?
- (d) Harish raises the stakes. Now Anirudh picks from 1 to 7, with

$$P(X = x) = \frac{x^2}{140}, \quad x \in \{1, 2, \dots, 7\}.$$

Compute the entropy. Then find the best yes/no strategy and its average number of questions. Why don't the two numbers match this time, when they did in the previous question?

Hint: Look at the probabilities in the previous question. What is special about them?

- (e) (*Bonus*) Which discrete probability distribution (for n outcomes) has the highest entropy? Could you prove it? What is the entropy?

Q 3.2. Cost Efficiency

Harish and Aarav want to send messages to each other. The only link they can afford is a copper wire owned by Aura, who charges \$4.00 for every 100 bits sent. The wire reaches any part of the globe, but outside interference and internal resistance distort the current more and more over long distances.

- (a) (*Bonus*) Why not use 26 different current levels, one for each letter, so that every letter costs a single pulse? What would you use instead, and why?
- (b) From now on, a low current is 0 and a high current is 1. If every letter of the English alphabet gets a code of the same length, how many bits does each letter need? How much does it cost to send the message ABRACADABRA?

Hint: How many yes/no questions does it take to single out one letter?

- (c) Aarav notices that the message uses only five letters, and some far more often than others. Design a code for these letters that gives frequent letters shorter codes. It must still decode correctly with no gaps or separators between codes.

Hint: If A is 0 and B is 01, what does 001 mean?

- (d) How many bits, and how much money, does your code from Question (c) save? Using the formula from Question 3.1(b), what is the least that *any* code could cost Harish?

§4 JoJo & Juice

Gyro Zeppeli and Johnny Joestar are participating in the Steel Ball Run (SBR) horse race. The map is a graph consisting of roads (edges), junctions (vertices), stages or sub-races (cycles in graph). Whenever they reach a junction, they get one bottle of juice but only once per junction. Gyro and Johnny travel each road exactly once, drinking one bottle per road. Additionally, they will get a bottle of juice whenever they clear a stage (Traverse all roads in that cycle).

Constraint 1: The defined starting and ending junctions are not part of any stage.

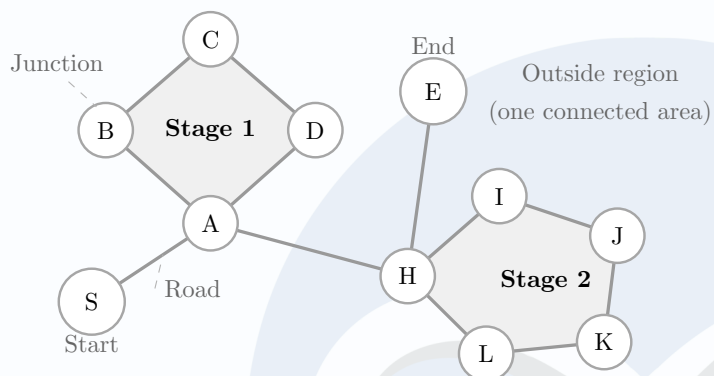
Constraint 2: A cycle in graph that is contractible (i.e, can be shrunk to a point) is a stage, otherwise not.

Constraint 3: A stage should not have any roads, junctions or loops inside it.

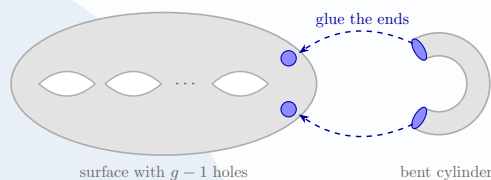
Constraint 4: The complement of the map forms one connected region, i.e, the “outside”.

- Q 4.1.** How many bottles of juice will Johnny have at the last junction of SBR, if he starts with 0 juice bottles at the starting junction (He obviously takes the juice at the starting junction)?

- Q 4.2.** Unfortunately, Gyro and Johnny were separated into 2 different batches of SBR. Their maps are completely different and don't have any common roads or junctions. But both have the same number of juice bottles at the end. Is it a coincidence?
- Q 4.3.** Imagine Earth is not a Sphere but a Cube. Now, solve 4.1.
(*Hint:* What is the difference between Sphere and Cube with respect to our question?)
- Q 4.4.** Imagine Earth is not a sphere but a donut (Formally, a torus). Now, do they always make a profit in all possible, completable layouts of the race? What is the minimum number of juice bottles they have to carry before starting the race so that they can complete the race without getting thirsty at the end in any possible layout?
- Q 4.5.** Now, Earth is a torus with g holes. Now solve 4.2.
(*Hint:* Take a donut with $g - 1$ holes and connect a bent cylinder in our current map to make it donut with g holes. How does this affect roads, junctions and loops?)



(a) An example race map



(b) Hint for Question 4.5



Damu damu damu damu!!!!

§5 Geometry Meets Numerical Analysis

The necessary definitions will be mentioned in the [addendum](#).

In an alternate universe, Ravi's brilliance brought him to the post of the village's divine protector. His fierce devotee, Siddha, was actually smart and loyal.

There was a major land tussle over the iridium-rich lands between Ravi and the ruthless industrialist, GK. Siddha was constantly plotting against GK to ensure Ravi stays in the lead.

Both sides were not fond of all-out wars, and so the tussle just remained a tussle and was an inconvenience for the people in the village. A stranded geologist decided a great way to settle the tussle. He devised a non-violent, mathematical duel for the territory, and it went as follows:

The disputed land was in the shape of an exact trapezoid $ABCD$, with parallel sides DC and AB . Let the two parallel bases have lengths $DC = a$ and $AB = b$, where $0 < a < b$, and height h . The challenge was to divide the trapezoid into two smaller trapezoids, using a line PQ parallel to the bases (with P on AD and Q on BC), such that the ratio of their areas was the same as the ratio between the lengths of their respective parallel bases. That is,

$$\text{ar}(DCQP) : \text{ar}(PQBA) = DC : AB = a : b.$$

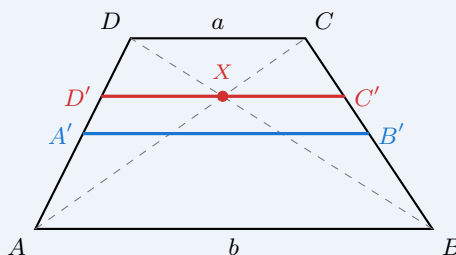


Figure 1: The trapezoid $T_0 = ABCD$, its midline $A'B'$, and the segment $D'C'$ through the intersection X of its diagonals.

Ravi was confident that he could win the duel, but he needed a strategy. Siddha immediately began working backwards, determined to find a construction that would leave GK with no chance.

Q 5.1. What should be the length of PQ such that $\text{ar}(DCQP) : \text{ar}(PQBA) = p : q$? Call that length $PQ_{p:q}$. What about the specific case when

- (a) $p : q = a : b$?
- (b) $p : q = 1 : 1$?

Siddha worked out the first construction and reported his findings to Ravi. But GK was not going to surrender the iridium-rich land so easily. He demanded that the next move be made without relying on guesswork.

Ravi challenged Siddha to find another horizontal slice, one that could be paired with the trapezoid's midline to produce a new trapezoid. Siddha began examining the intersection of the diagonals.

- Q 5.2.** Let X be the intersection of the diagonals AC and BD , and let $D'C'$ be the segment through X parallel to the bases, with D' on AD and C' on BC (Figure 1). What is the length of $D'C'$? Call it PQ^* .

With both slices in hand, Ravi now had a strategy. Siddha paired the midline with the new slice to obtain a new trapezoid T_1 . Ravi suspected that although the shape changed with every move, some quantity might remain untouched throughout the process.

- Q 5.3.** Let $A'B'$ be the midline of $T_0 = ABCD$, i.e. the segment joining the midpoint A' of AD to the midpoint B' of BC . The segments $A'B'$ and $D'C'$ are the parallel sides of a new trapezoid $T_1 = A'B'C'D'$ (Figure 1). Establish an invariant (some quantity/feature that doesn't change) between T_0 and T_1 .

Siddha soon uncovered the invariant and brought the result to Ravi. The discovery gave Ravi hope: perhaps he could settle the dispute without constructing every trapezoid one by one.

GK, however, was growing impatient. He wanted to know whether this process would ever end, or whether Ravi's victory would remain forever just out of reach.

- Q 5.4.** Ravi turned to the sequence of trapezoids and asked Siddha to investigate what happened when the construction was repeated indefinitely: applying the same construction to T_1 (its midline, and the slice through the intersection of its diagonals) gives T_2 , and so on. Siddha wonders, "If T_i is the trapezoid I obtain after i such steps, what happens if I keep recursing and $i \rightarrow \infty$?"

The stranded geologist then revealed that Ravi's geometric duel had a surprising connection to numerical analysis. A repeated geometric construction could be turned into a recursion scheme, allowing the answer to be approximated computationally.

Ravi saw an opportunity to finish the duel once and for all. If Siddha could express the process as a recurrence relation, they might find the desired mean without having to draw infinitely many trapezoids. A recursive scheme of the form

$$x_{k+1} = f(x_k)$$

is very popular in root finding. If f is continuous and the iterates converge, their limit $x^* = \lim_{k \rightarrow \infty} x_k$ satisfies $f(x^*) = x^*$, i.e. it is a root of $f(x) - x$.

Siddha realised that the invariant from the previous question could be the key. Ravi challenged him to turn the geometric strategy into a numerical one.

- Q 5.5.** Let x_k be the length of the longer parallel side of T_k (so $x_0 = b$). Set up a recursion scheme of the form $x_{k+1} = f(x_k)$ whose iterates converge to the GM of a and b . What is $f(x)$?
(Hint: The answer to Q 5.3 would help!)

With the recurrence established, Ravi was almost ready to claim victory. But he wanted a method that could be executed by a computer, repeatedly refining the approximation until the answer became clear. Siddha took up the final challenge.

- Q 5.6.** (Bonus) Write a program (in any language) to perform this numerical approximation, stopping once successive iterates agree to within a chosen tolerance.

§6 A Spherical Odyssey

Refer to the **addendum** for the conventions used. Read the story carefully to complete the journey.

Prajithsseus and his coordinator crew win the Troy Mathematics Championship, a victory that took just short of 10 years :D. They set sail for home.

$$\text{Troy } T = (40^\circ\text{N}, 26^\circ\text{E}), \quad \text{Ithaca } I = (38^\circ\text{N}, 20^\circ\text{E}).$$

Their chief navigator **Kailachus** charts the course using the stars and the geometry of the Earth. Yet even his calculations are no match for the wrath of Poseidon.

Part I: The Curse of the Wind: The crew steers straight for Ithaca, in the initial bearing of the great circle $T \rightarrow I$. But the wind blowing towards the east pushes them, and their actual track ends up rotated 30° to the *left* of that direction. After 300 km on this track they land on an island C , home of the Cyclops Polyphemus, whom Prajithsseus blinds before escaping.

Q 6.1. (The curse of the wind)

- Find the great-circle distance and the initial direction from T to I .
- Using the 30° deviation and $v_s = 8$ km/h, find the wind-drift speed u .
- Find the exact coordinates of Cyclops' island, C .

Part II: Ts was NOT the wind gng. Still on the same track, the crew reaches the island of Aeolus, A , who gives them a bag that calms Poseidon's wind while it stays sealed. Despite **Satvik's** warnings, **Anirudh** opens it, and the storm winds blow the ship back to A . Poseidon's westerly wind returns. From A , Ithaca lies exactly north-west, in direction 315° . The crew again steers straight for Ithaca, in the same direction as $A \rightarrow I$, and follows the resulting path to Circe's island C_e .

Q 6.2. (Circe's island) Find the direction of the ground track from A . By how many degrees, and to which side, does it differ from the direction in which the crew steers?

Part III: Sirens and a Choice. Circe sends the crew to the Underworld, where Tiresias warns Prajithsseus not to touch the cattle of the sun god Helios. **Agamenachalam** and **Eppalles**, former comrades from Troy, add warnings of their own. Back at C_e , the crew sails along the great circle $C_e \rightarrow I$ (adjusting their heading to stay on it), past the Sirens, to a strait with Scylla (a six-headed monster) on one bank and Charybdis (a ship-swallowing whirlpool) on the other.

Q 6.3. (Scylla or Charybdis?) The strait is $w = 1000$ m wide. Let x be the distance (in metres) of the ship's path from Scylla's cliff, $0 \leq x \leq w$. With 30 crew members aboard, the expected number of men lost is

$$L(x) = \underbrace{6e^{-x/200}}_{\text{Scylla}} + \underbrace{30e^{-(1000-x)/250}}_{\text{Charybdis: ship and crew}}.$$

Find the x that minimises L , prove it is a global minimum, and state which monster Prajithsseus should steer toward. Compare $L(x^*)$ with $L(500)$.

Part IV: The Giant Moostake. Nearing Ithaca, the starving crew lands on Thrinacia, the island of the Sun (**RaviK**), and slaughters Helios's cattle despite every warning. The moment they set sail from

$Th = (38^\circ\text{N}, 20.5^\circ\text{E})$, Zeus strikes the ship with a thunderbolt. Everyone dies except Prajithsseus, who clings to a small raft with no sail and no oars. Poseidon's easterly wind is still blowing, and nobody is steering, so the raft just drifts toward local east for 9 days until it washes up on Calypso's island K . (Calypso does *not* fall in love with him.)

With some help from the goddess of wisdom and war, Athena, he finally, finally reaches Ithaca.

Q 6.4. (Calypso's island (the raft))

- (a) What is the raft's velocity over the ground, and how far does it drift in 9 days?
- (b) The raft's path is a circle of latitude. Find its radius and hence find the exact coordinates of Calypso's island K .

Q 6.5. (Bonus: The Voyage of Prajithsseus)

- (a) Draw a labelled map of the voyage showing T , C , Th , K and I . Show the great-circle routes (as modelled) and the raft's path along a circle of latitude differently. Mark the relevant directions and the direction of wind drift.
- (b) What are the mathematical issues with the modelling in this problem? List all that you can think of. Any further analysis will receive more points!
- (c) **Rhumb Line** Consider only the first part of the journey $T \rightarrow C$. Suppose the crew holds the compass heading θ_0 (the initial direction of $T \rightarrow I$, found in Problem 6.1(a)) at speed v_s , in the easterly wind of speed u found in Problem 6.1(b).
 - a) Show that in the local (east, north) frame the ground velocity is constant. Deduce that the true ground track has a *constant direction* $\beta = \theta_0 - 30^\circ$.
 - b) Let $\varphi(t)$ and $\lambda(t)$ be the ship's latitude and longitude. Write $\dot{\varphi}$ and $\dot{\lambda}$ in terms of the ground velocity and R , and show that

$$\frac{d\lambda}{d\varphi} = \tan \beta \sec \varphi.$$

Then find λ as a function of φ .

- c) Show that the distance travelled along the track while the latitude changes from φ_0 to φ is $s = R(\varphi - \varphi_0)/\cos \beta$.
- d) Find the exact position of the ship after 300 km along this true track, and compare it with your coordinates of C from Problem 6.1(c). By how many kilometres do they differ?
- e) Estimate how this difference changes if the distance is doubled, and use your answer to comment on how good the great-circle idealisation is for the longer parts of this voyage.

§7 It's finally done bro



Da goat wishes da luck to all da aspirants.

