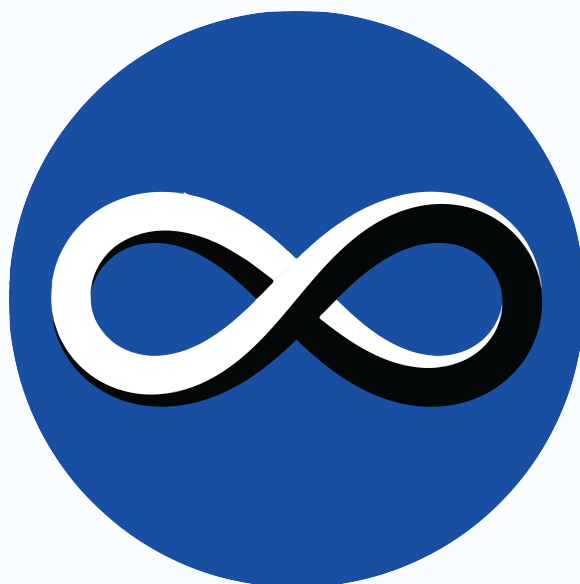


# MATHEMATICS GUILD APPLICATION

2025-26



MATHEMATICS CLUB



TECH CONTINGENT  
IIT MADRAS

## Instructions

### General Instructions

- Mention the following details in the start of your application.

Name:	Insti Name:	
Roll:	Room:	Hostel:
Phone (WA):	CGPA:	Email:

- Join the Aspiring Guild Members WhatsApp group for further updates: [Aspiring Math Guild Members Whatsapp Group](#).
- The recommended font is a standard font size 11-13. Handwritten answers may also be submitted.
- The applications have to be submitted in PDF format, named as:  
    <First\_name>\_<Roll\_Number>\_Mathematics\_Guild.pdf  
    . For example, Pranjal\_EE23B180\_Mathematics\_Guild.pdf.
- You can upload the finished applications in this [Google Form](#).
- You may submit the completed application on or before **11:59 PM, 04/06/2025**.

### Note:

- It is fine even if you don't answer all the questions.
- Focus on the first few questions before attempting the bonus questions.
- If you have any queries, you can reach out to the Guild Captains anytime you want:
  - Pranjal Varshney (EE23B180): [7780661602](#)
  - Pratyaksh Jain (EE23B057): [9686872542](#)

## §1 General Questions

1. Why do you want to be a part of Mathematics Guild and what is your motivation for the same? What makes you a suitable Guild member (mention relevant skills and experience)?
2. Mention a few (minimum 2) topics of Mathematics you believe you are skilled at (or have a bit of experience) and a few others you wish to improve upon.
3. Have you attended any Mathematics Competitions / Olympiads before? If so, mention all of them. Name all the Mathematics Competitions you think we should participate in as a Guild.
4. As a guild we explore new concepts every week and solve problems. How many hours per week will you be able to give to the Guild. Mention all PoRs/activities you will be involved in and the time you will give to each of them.
5. Mention your availability in the following time periods:
  - a) **On the Day of SMMC:** This year SMMC is held on 11th October (Saturday) and lies in the middle of quizzes. Specifically its before the last quiz (D-slot) on Monday. (Don't worry, we won't officially prepare for it during the quizzes, but you and your teammate are always welcome to use any free time during quizzes to prepare for it)
  - b) **In December during the winter vacation:** Two important competitions for the Guild (Inter-IIT and CMI STEMS) fall in the winter break. Would you be willing to stay back for a fraction of the vacation?

Note that this is not a metric for your selections but simply a glimpse into your tenure. We understand that no Guild member can be free throughout the year. This is to help us balance and rotate members for competitions.

## §2 Number Theory

“Mathematics is the queen of the sciences — and number theory is the queen of mathematics.”

— Carl Friedrich Gauss

### 1. Quartic Harmony

Find all integer tuples  $(a, b, c, d)$  that satisfy the equation :

$$a^2 + b^2 + c^2 + d^2 = 2abcd.$$

### 2. Permutation Displacement

Let  $m$  be a positive integer. Denote by  $\sigma$  any ordering of the set  $\{1, 2, \dots, m\}$ . We define the cost of an ordering  $\sigma$  by

$$C(\sigma) = \sum_{j=1}^m |\sigma(j) - j|.$$

Determine, as a function of  $m$ , the number of *distinct* values that the quantity  $C(\sigma)$  can attain as  $\sigma$  ranges over all  $m!$  permutations of  $\{1, 2, \dots, m\}$ . Condense your function (if not already done) to be valid uniformly for all positive integers  $m$ , without relying on separate cases.

### 3. Multiply and conquer!

Find all positive integers  $k < 526$  for which there exists a positive integer  $n$  such that:

$$\left\{ \frac{n}{526} \right\} + \left\{ \frac{2n}{526} \right\} + \dots + \left\{ \frac{kn}{526} \right\} = \frac{k}{2}$$

where  $\{x\}$  denotes fractional part of  $x$ .

## §3 Calculus

*“There are only  $\int_0^1 \left( \frac{52x^{7/2} - 66x^{5/2} + 22x^{3/2}}{\sqrt{x}} \right) dx$  kinds of people in the world: Those who know*

*Calculus and those who don’t.”*

#### 1. Trivial exponentials

Determine all pairs  $(a, b)$  of real numbers that maximise the integral

$$\int_a^b e^{\cos x} (380 - x - x^2) dx$$

#### 2. A Beautiful Golden Integral

Let  $\phi$  be the positive root of the equation

$$x^2 = x + 1.$$

Evaluate the integral

$$\int_0^n \left[ \phi^{\lfloor x \rfloor} \right] dx.$$

Report your answer in an expression involving Fibonacci numbers.

*Hint:* Try to think of the Fibonacci numbers and their properties.

#### 3. Dysfunctional equation

You are given a function  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  such that  $f(f(a) - b) + bf(2a)$  is a perfect square for all integers  $a$  and  $b$ .

- Find the value of  $f(0)$
- Find all functions  $f$  which satisfy the equation.

## §4 Geometry

*“What’s the best way to woo a mathematician?” Use acute angle... :)*

#### 1. Let your mind rest in this one

$ABCD$  is a parallelogram. It has been given to you that:

- $\angle D = 60^\circ$
- $AB = \sqrt{3} + 1$
- $AD = 2$

$AD$  has its midpoint at  $M$  and the angle bisector of  $C$  meets  $BM$  at  $K$ . What is the angle  $\angle CKB$ ?

## 2. Get to the point

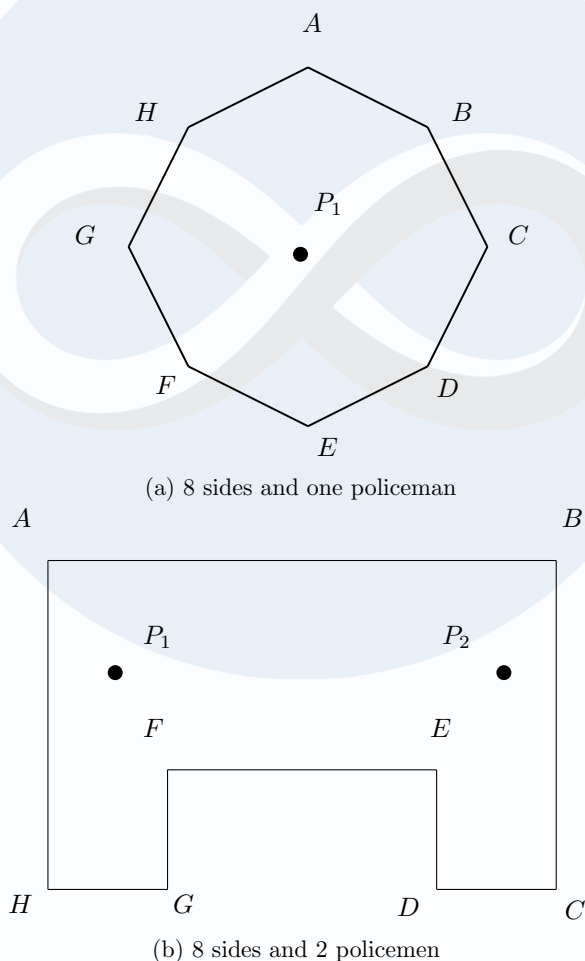
Triangle  $A_1A_2A_3$  has a right angle at  $A_3$ . A sequence of points is now defined by the following iterative process, where  $n$  is a positive integer. From  $A_n (n \geq 3)$ , a perpendicular line is drawn to meet  $A_{n-2}A_{n-1}$  at  $A_{n+1}$ .

- Prove that if this process is continued indefinitely, then there exists one and only one point  $P$  which is interior to every triangle  $A_{n-2}A_{n-1}A_n, n \geq 3$ .
- Let  $A_1$  and  $A_3$  be fixed points. By considering all possible locations of  $A_2$  on the plane, find the locus of  $P$ .

## 3. FBI open up

A robber is hiding in his house, trying to avoid getting caught. The police plan to break the door and find him. To their convenience, there are no objects in the house to block their view, just walls. The only catch is... it's an irregularly shaped house with 36 vertices.

The police must ensure that the entire house is under their field of view (assume every policeman has  $360^\circ$  field of view). For example:



The cops can place themselves wherever they like once they get in, and they know the layout of the house once they get there (though the number of vertices is known beforehand).

What is the smallest number of cops you need to ensure you can cover the whole house? Explain the reasoning behind your answer.

(*Hint:* You can follow a general idea for any house with  $3n$  vertices.)

## §5 Combinatorics & Probability

*“The very term ‘combinatorial methods’ has an oxymoronic character.”*

### 1. Lets count!

- a) How many 6-digit numbers whose digits are from 1 to 5 only, have every digit repeated at least once, i.e., each digit should appear at least twice in the 6-digit number?
- b) (An independent question, i.e., the conditions in the previous question don't apply) Pranjal was bored so he started listing down 6-digit numbers, but with a twist. The digits in each number he wrote down were in **non-increasing order**. He listed down these numbers in **increasing order**. Now, he gives you a challenge. What is the 2025<sup>th</sup> number in the list he prepared?
- c) Soon as you were done manually writing down 2025 numbers ;), Pratyaksh came up to you and gave you another challenge: Find the number of positive integers less than or equal to the answer to part (b) whose base-7 representation contains no digit equal to 0.

### 2. A fierce WAR

The two mighty kings, Navin of the North and KK of the South are at war! Each has dispatched 1000 of their finest warriors: Navin's clad in jet-black armor, KK's in crimson. To observe the unfolding chaos, the kings send their sharpest minds, Pranjal and Pratyaksh, to view the battlefield from the side, where the warriors appear arranged in a single line.

At a certain moment, they are asked to count the number of **blocks**: groups of consecutive soldiers from the same side. For example, in the sequence **BBBBRRRBBRBB**, there are 5 blocks: **BBBB**, **RRR**, **BB**, **R**, and **BB**.

But the battlefield is in constant flux — the warriors continuously shift positions, and so the number of blocks keeps changing. To address this, Pranjal and Pratyaksh decide to compute the **average number of blocks over time**. Unfortunately, their final report is lost in transmission, leaving the kings furious.

Can you help calm their fury by uncovering the answer the kings never received? **Assume no soldier ever dies**

## §6 It's Revenge time

*"The student has become the master."*

— Darth Vader

Use the experience gained from the above questions to challenge us with 3 questions from any domain of your interest.

- The questions can be of any difficulty level.
- You are free to take inspiration from Olympiads or other sources on the internet but do not directly copy questions.
- Assume you are preparing a Problem set for the Guild, i.e you don't want us to google the questions and come up with a solution.

## §7 Bonus

This is a **Bonus section**. Once you are done with the above problems, you can try these out.

### 1. When you've scrolled too much Instagram!

Shown below is *Bombardino Crocodilo*. He is playing a game with *Tung Tung Tung Sahur*. They take chances to draw a min of 1 and a max of 3 coins from 21 coins. Whoever draws the last coin loses. If *Sahur* starts first, who wins and with what strategy, if both play optimally.



Figure 2: Bombardino Crocodilo

### 2. Smooth Operation

Let  $S$  be a set and  $\&$  be a **Binary operation** defined on  $S$  satisfying the laws:

$$\text{a) } x \& (x \& y) = y \quad \forall x, y \in S$$

$$\text{b) } (y \& x) \& x = y \quad \forall x, y \in S$$

Show that  $\&$  is a commutative operation but not necessarily associative.

### 3. comPelled to give up

Find the smallest positive integer solutions to  $x^2 - 31y^2 = 1$ .