Srinivasa Ramanujan Mathematics Competition

2025

Mathematics Club × **Maths and Physics Club**
IIT Madras **IIT Bombay**

28$^{\text{th}}$ September 2025

Instructions:

- The duration of the exam is **3 hours**.
- Start the solution to each question on a **new sheet of paper**.
- Write the solution to each question in an **uninterrupted** manner. That is, don't write solutions to other questions in between the solution to the current question.
- Indicate the question number on the **top-left corner of each page** (both sides) in the format: **P<question_number>**.
  For example, pages containing any part of Question 1 should have **P1**.
- The only identifier on your answer sheet should be your **SRMC Roll Number**. Do **NOT** write your name or any personal information. Indicate your roll number on the **top-right corner of each page** (both sides).
- This is a **closed-book, closed-internet test**. Electronic devices and calculators are strictly prohibited.
- Each question carries **10 points**.
- There is **no negative marking**.

Roll Number: | | | |

The Mathematics Club of IIT Madras and the Maths and Physics Club of IIT Bombay envisioned a pan-Indian collegiate mathematics olympiad. Through months of unwavering dedication, their efforts culminated in the Srinivasa Ramanujan Mathematics Competition, a celebration of mathematical brilliance across the nation.

1. Prove that for all $x, y, z, w \in \mathbb{R}^+$ the following inequality holds:

$$\frac{xy + yz + zw}{x^2 + y^2 + z^2 + w^2} + \frac{xyz + yzw}{x^3 + y^3 + z^3 + w^3} + \frac{xyzw}{x^4 + y^4 + z^4 + w^4} < \frac{2 + \sqrt{5}}{4} + \frac{\sqrt[3]{4}}{3}$$

2. Navin is studying some sets contained in the collection $\mathbb{N}_{1000}$, that is, the collection of sets $N \subset \mathbb{N}$ ($\mathbb{N}$ denotes $\{1, 2, 3, \dots\}$) with the size $|N| = 1000$. He is interested in sets with the following special property which he calls *k-stiffness*. A set $N \in \mathbb{N}_{1000}$ is called a *k-stiff set* if every k-element subset of N has the same least common multiple L_N.

   a) Given an arbitrary 20-*stiff* set, what is the the maximum possible guaranteed number of pairs of co-prime elements contained in it?

   b) Across all 20-*stiff* sets, find the maximum possible number of pairwise co-prime elements contained in them.

3. Prove the following inequality for all $y \in \mathbb{R}^+$:

$$\int_0^1 \sqrt{x - x^{e^y}}\, dx \leq \frac{\sqrt{y}}{2}$$

   where e is Euler's constant.

4. Let $N = 2025^{2025}$. $S(X)$ denotes the sum of digits of the number X written in base 10. Let the integer $A = S(S(S(N)))$. Find the smallest integer B (≥ 2) such that

$$(101)_B = (aabbcc)_A$$

   for some $a \in \{1, ..., A - 1\}$ and $b, c \in \{0, ..., A - 1\}$.

   (Note: $(xyz)_W$ represents a number when written in base W has digits xyz)

5. Shivansh is very interested in primes, so he decides to multiply the first 4 prime numbers to get the product 210. Now he ponders over some interesting questions related to the subsets of the integers written from 1 to 210 (both inclusive).

   a) How many subsets are there, where the number of elements is a multiple of 3 and the sum of elements is a multiple of 5?

   b) After getting the answer to his first question, he also wonders about the same for subsets having an even number of elements with the sum of elements being a multiple of 7; but he only wants to know if the number of such subsets is more than the one obtained previously. Are the number of such subsets less or more?

6. You are given three non-collinear marked points A, B, C in the Euclidean plane. You are equipped with the following tools:

   - ***Tool 1.*** Given two points as the endpoints of the major (or minor) axis and a third point on the ellipse, draw the corresponding ellipse. In addition, *Tool 1* returns the value
   $$\frac{\left(\text{Area}(E \cap C)\right)^2}{\text{Area}(C)\,\text{Area}(E)}$$
   where C is the circumcircle of $\triangle ABC$ and E is the ellipse drawn.
   - ***Tool 2.*** Given two lines ℓ_1, ℓ_2, construct another line ℓ_3 such that ℓ_1 is the angle bisector of ℓ_2 and ℓ_3.
   - ***Tool 3.*** Join two marked points with a line, or mark the intersection of two given lines.

   Assume that *Tool 1* returns the value only to a finite precision. Using these three tools, construct an ellipse for which the value returned by *Tool 1* is 1.

7. After solving the ruin's triangle puzzle, Rhythm hears a rumble. A hidden passage opens. Dhruva waits inside pointing to four constellations namely, Orion, Canis Major, Taurus and Auriga. Each of them have nine stars indexed from 1 to 9. Dhruva claims 18 stars, making them shine brightly. The remaining 18 become dim and now belong to Rhythm.

   The game starts lasting a total of 18 rounds. In each round, Dhruva announces one of his bright stars. Rhythm must respond with one of his dim stars. Rhythm wins the round iff he responds with a twin star (i.e. it is in the same constellation or has the same index as Dhruva's star). Stars are discarded after each round once they are announced.

   What is the maximum number of rounds Rhythm is guaranteed to win, no matter what Dhruva selects?

You have reached the end of the question paper.

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