Integration Bee Round 1

MATHEMATICS CLUB IITM

13 SEPTEMBER 2025

---

1. The paper consists of **20 questions** and the total duration is **40 minutes**.
2. Participants must write their **final answers** clearly in the space provided on the answer sheet. Rough work may be done on the question paper, but it will not be evaluated. Only the answers written in the designated spaces will be considered for marking.
3. The use of calculators, mobile phones, or any online resources is **strictly prohibited**. Any violation of rules will result in immediate disqualification.

---

1. Here we go.

   If $g(x)$ satisfies $g(x) + g(x+2) = 10 \quad \forall x \in \mathbb{R}$ then what is $\displaystyle\int_{-1}^{7} g(x)\,\mathrm{d}x$

2. Do we like floors?

   $\displaystyle\int_0^1 x \left\lfloor \frac{1}{x} \right\rfloor \mathrm{d}x$

3. Yes we do!

   $\displaystyle\int_0^1 \left\lfloor \sqrt{1+\frac{1}{x}} \right\rfloor \mathrm{d}x$

4. Ceil felt left out :(

   $\displaystyle\int_1^\infty \frac{\lceil x \rceil}{x^2 2^{\lfloor x \rfloor}}\,\mathrm{d}x$

5. Impartial fractions

   $\displaystyle\int_0^{\ln(2)} \left\{ \frac{1}{e^x - 1} \right\} \mathrm{d}x$

6. Those functions left the party.

   $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\ln(\cot x)}{((\sin x)^{2009} + (\cos x)^{2009})^2} (\sin 2x)^{2008}\,\mathrm{d}x$

7. Not trigonometry.

   $\displaystyle\int_0^1 \cos^2(\ln x)\,\mathrm{d}x$

8. Definitely not trigonometry.

   $\displaystyle\int_0^3 \frac{4e^{2x} - e^{3x}}{e^{2x} + e^{3x}}\,\mathrm{d}x$

9. Trigonometry.

   $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sin x + \cos x}\,\mathrm{d}x$

Please turn over

10. More trigonometry.

$$\int_0^1 \tan^{-1}\left(\frac{x}{1-x}\right) dx$$

11. Mostest trigonometriest.

$$\int_0^1 \exp\left(x - a\tan(e^x)\right) \cdot \sec^2(e^x) da$$

12. Partial beta functions?

$$\int_0^5 \frac{x}{\sqrt{16-3x}}\,\mathrm{d}x$$

13. A trivial integral!

$$\int_0^{\frac{\pi}{4}} \frac{1}{\displaystyle\sum_{n=0}^{\infty} \frac{x^{4n}}{(2n)!^2} + 2\sum\sum_{0\le n<m<\infty} (-1)^{n+m} \frac{x^{2n}}{(2n)!}\frac{x^{2m}}{(2m)!}} dx$$

14. A nice number...

$$\int_0^1 2^{\sqrt{x}} \ln^2(2) + \ln^2(1+x)\, dx$$

15. BeComplex®

$$\oint_{|z|=3} \frac{z^2-3}{(z-1)(z^2+1)} dz, z \in \mathbb{C}$$

16. A positive nest

$$\int_{-2025}^{2025} \underbrace{|||||x|-1|-1|-1|-\cdots|}_{2025\text{ (-1)'s}}\,\mathrm{d}x$$

17. TanTaTan...TanTaTan...

$$\lim_{n\to\infty} \int_{-\infty}^{\infty} \tan\left(\frac{1}{2}\tan\left(\frac{1}{2}\tan\left(\frac{1}{2}\left(\frac{1}{1+x^{2n}}\right)\right)\right)\right) dx$$

18. My heart yearns...

... to find the length of the curve $r = 1 - \sin\theta$ where $0 \le \theta \le 2\pi$

19. And here we part.

$$\int_0^{\infty} \frac{1}{\sqrt{x^3+1}} dx + \int_{\infty}^{0} \frac{1}{\sqrt{x}+x^2} dx$$

20. If you find this paper easy and want a challenge, find A+B:

$$A = \left\lfloor \frac{\displaystyle\int_0^1 \sum_{n=1}^{\infty} \left(\left(\frac{1}{n^2}\int_0^1 \left(\exp\left(\frac{x}{n}\right)+n\right)\mathrm{d}x\right)\right)\mathrm{d}y}{\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}} \right\rfloor$$

$$B = \sin\left(\frac{\pi}{8}\right)\int_0^{\infty} \frac{1}{1+x^{5+A}} dx$$

🐝 ***HAPPY BUZZIN!*** 🐝