Srinivasa Ramanujan Mathematics Competition

SRMC 2024-25

IIT Madras × IIT Bombay

19th January 2025

Instructions:

- The duration of the exam is **3 hours**.
- Start the solution to each question on a **new sheet of paper**.
- Write the solution to each question in an **uninterrupted** manner. That is, don't write solutions to other questions in between the solution to the current question.
- Indicate the question number on the **top-left corner of each page** (both sides) in the format: **P<question_number>**.
  For example, pages containing any part of Question 1 should have **P1**.
- The only identifier on your answer sheet should be your **SRMC Roll Number**. Do **NOT** write your name or any personal information.
- This is a **closed-book, closed-internet test**. Electronic devices and calculators are strictly prohibited.
- Each question carries **10 points**.
- There is **no negative marking**.

SRMC Roll Number : _____

The Mathematics Club of IIT Madras and the Maths and Physics Club of IIT Bombay envisioned a pan-Indian collegiate mathematics olympiad. Through months of unwavering dedication, their efforts culminated in the inaugural Srinivasa Ramanujan Mathematics Competition, a celebration of mathematical brilliance across the nation.

1. There are n red points and n blue points in a plane, with no three points collinear. Find all the possible values of n for which n segments can be drawn, each segment joining a distinct red point and a distinct blue point, such that no pair of segments intersect.

2. Find, for $x, y, z \in \mathbb{R}^+$, the minimum value of the following expression.

$$\frac{x+3z}{x+2y+z} + \frac{4y}{x+y+2z} - \frac{8z}{x+y+3z}$$

3. Let ABC be a triangle 'rsect CA and AB at E and F respectively. Prove that the perpendicular to BC at D is a diameter of the circumcircle of AFE.

4. Let φ be Euler's totient function with $\varphi(1) = 1$. Find

$$\lim_{n\to\infty} \frac{\prod_{k=1}^{n} \pi^{\varphi(k)}}{\prod_{k=2}^{n} \left(\pi^k - 1\right)^{\frac{\varphi(k)}{k}}}$$

5. A cube has 1 red and 5 white faces. We place it on a table so that it rests on the red face. An "edge move" is a 90° rotation of the cube around one of the four edges resting on the table, so that a new face will then be resting on the table. Suppose somebody performs 12 edge moves, such that each one of the cube's 12 edges is used exactly once as the axis of rotation for an "edge move". Prove that after the final move, the cube will again rest on the red face.

6. Find all pairs of non-negative integers x and y that satisfy the equation

$$p^x - y^p = 1$$

where p is an odd prime.

7. Show that

$$\prod_{i=1}^{2025} x_i = 2^{2024} \sum_{i=1}^{2025} x_i$$

where $x_i \in \mathbb{N}$ for all i, has only finitely many solutions.