Meth Recipe Book

for DC Application

2025-26

Mathematics Club

Centre for Innovation
IIT Madras

Some Encouragement from our lovely heads

Nabin with tie, wishing you all the best

“You got this! Reverse belt my coords in interview” - KK (not the car)

DC Application Addendum

Mathematics Club

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§1 Infinite Possiblities

What is a Random Variable?

In the most general and mathematical sense, a **Random Variable** is *a measurable function from a probability space into a measurable space*.
In simpler terms, it is a rule that assigns a numerical or structured value (often real-valued) to each possible outcome of a random experiment, in a way that allows probabilities to be meaningfully defined for events like "$X \le x$" or "$X \in A$".

CDF – Cumulative Distribution Function

In many probabilistic models, we are often more interested in the probability that a random variable lies within a certain range rather than taking a specific value. This is because such probabilities are crucial in tasks like prediction, estimation, and handling uncertainty (for example, determining tolerance levels in noisy systems).

We therefore define the **Cumulative Distribution Function (CDF)** of a random variable X as

$$F_X(x) = \Pr(X \le x).$$

For a continuous random variable with probability density function $f_X(x)$, this becomes

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\, \mathrm{d}t.$$

Some more cool things in Probability

Having PDFs and CDFs gives us a way to describe how probabilities are distributed – but that's not the whole story. In practice, we often need to summarise or quantify these distributions. That's where concepts like Expectation, Variance, and others come in.
For some unknown reason, I wanted to put this...

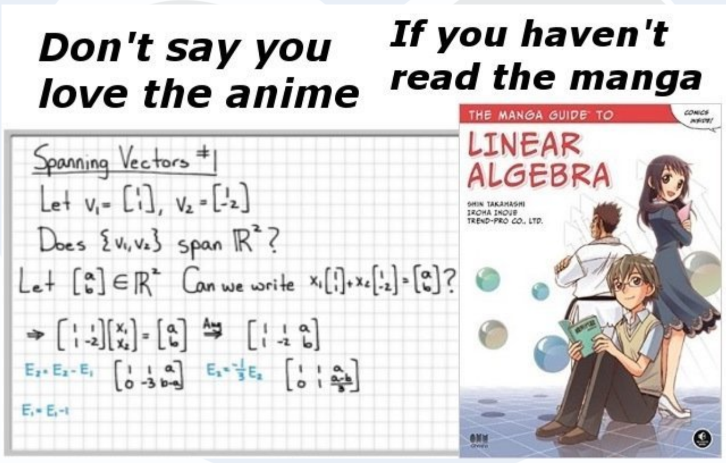
$$\int_{-\infty}^{\infty} \sin(x)\,\mathrm{d}x \quad \to \text{DNE}$$
$$\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,\mathrm{d}x \quad \to \text{DNE}$$

...this is probably useless...right?

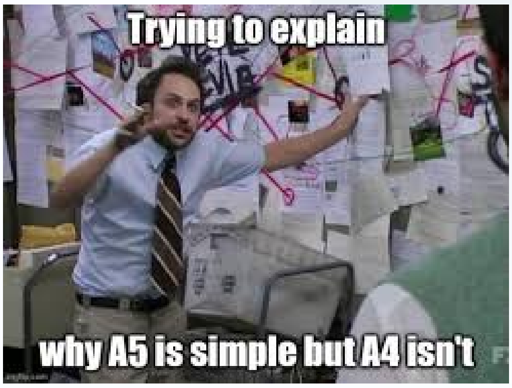
§2 Linear Algebra

Resources:

- Essence of linear algebra - 3Blue1Brown
- Linear Algebra by Gilbert Strang, 4th ed.
- MIT 18.06 Linear Algebra, Spring 2005
- The applications of eigenvectors and eigenvalues



§3 What do you call a Group of Mathematicians?



Set theory forms the fundamental basis of mathematics. The reason they are so ubiquitous is that they are simple constructs that can be used to formalise many other objects in mathematics. Now sets are good, but what if you wanted MORE power?

Groups are a natural extension of sets. They gain a binary operation at the cost of the ubiquity of sets. Gaining the operation, however, is extremely powerful - granting structure, symmetry, and algebraic order. Group operations, in turn, need to follow certain properties called group axioms.

Suggested reading

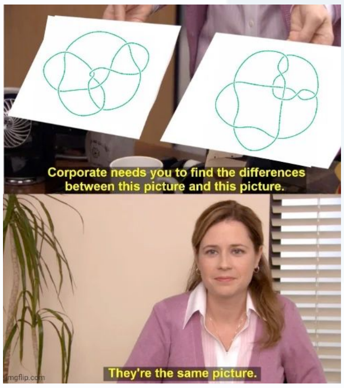
- Contemporary Abstract Algebra - JA Gallian (just the first couple of chapters should be enough)

This section is pretty simple; the results are somewhat easy to prove, but we want to see how well you can grasp the basics of a new topic, and your proof-writing skills :)

Some other interesting stuff

1. **Burnside's Lemma:** This is a really nice connection from group theory to combinatorics
2. **Lie Groups:** Some groups can have additional structure from being studied in other areas of math. Structure like a topological space or differentiable manifold. Lie Groups are groups that are also differentiable manifolds and are a very deep subject, which eventually broke off from group theory as its own branch of math. ***A warning, though this is an insanely deep and complicated rabbit hole***
3. If you understood the above meme and its importance, put it in your app somewhere; we'll be very impressed.

§4 To be or Knot to be?



§4.1 The Core Mathematical Problem

Definition of a Knot

A *mathematical knot* is a closed curve (like a circle) embedded in three-dimensional space that does not intersect itself. The critical distinction from common knots is that the ends are joined, making it impossible to untie.

The Equivalence Problem

The central challenge in knot theory is determining whether two different-looking knot projections are actually the same knot - that is, whether one can be smoothly deformed into the other without cutting or passing a strand through itself.

Reidemeister Moves

There are three fundamental moves, known as **Reidemeister Moves**, that can transform any diagram of a knot into another diagram of the same knot:

(i) **Twist/Untwist (R_1):** Adding or removing a simple twist.

(ii) **Poke/Slide (R_2):** Moving a strand over another strand.

(iii) **Pass (R_3):** Moving a strand over or under a crossing.

Rotation of knot in space is NOT a Reidemeister move.

Knot Invariants

Since proving that two knots are the same is easy (by demonstrating the above moves), the real challenge lies in proving that they are *different*.

Knot invariants are properties that remain unchanged under any Reidemeister move. Some key invariants include:

- **Tri-colourability :** A basic invariant used to prove that the Trefoil knot is fundamentally different from the Unknot.
- **Knot Polynomials:** Advanced invariants that assign algebraic expressions to knots, serving as unique "fingerprints" to distinguish between complex or visually similar knots. Examples include the Alexander polynomial, Jones polynomial, and HOMFLY polynomial.

§4.2 Some Interesting Applications

Knot theory is far from purely abstract; it plays an essential role in the life sciences.

- **DNA Topology and Enzymes**
  DNA, particularly in bacteria, exists as a long, knotted loop. During replication, these strands become highly tangled.

  Topoisomerase enzymes are biological machines that manage DNA topology. They function by temporarily cutting the DNA strand, passing another strand through the gap, and re-sealing the cut to unlink or unknot the molecule.

- **Protein Structure**
  A small but significant number of proteins have knots in their structure. Knot theory helps researchers understand their folding mechanisms and how mis-knotting can lead to protein malfunction.

- **The Shoelace Problem**
  A familiar observation is that there are two ways to tie the final bow in a shoelace, resulting in either the **Square Knot** or the **Granny Knot**.
  Knot theory explains that these are the connected sums of the Trefoil knot and its mirror image, which determines their stability and tendency to loosen.

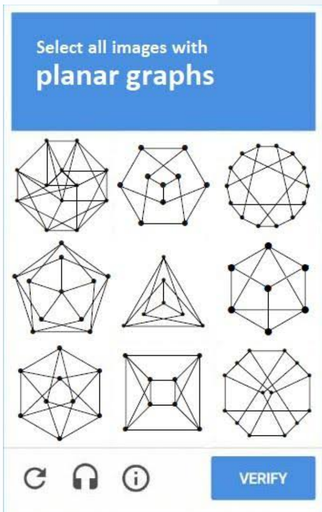
- **Tangle Physics**
  The principles of knot theory explain the physics of why cords and cables tend to self-tangle (e.g., in a drawer). Techniques such as the **bi-fold coil** can restrict the movements necessary for knot formation.

- **Synthetic Chemistry** Chemists apply knot theory to synthesize molecular knots—tiny molecules that are chemically knotted, such as a trefoil-knotted molecule. These structures possess unique properties due to the topological restrictions on their atoms.

TL;DR

For a better understanding of knot theory, you may go through this video and this video as well.

§5 Help Me Colour



§5.1 Basic Definitions needed

- **Simple Graph:** A graph with no loops or parallel edges between the same pair of vertices.
- **Tree :** A tree is a connected acyclic graph.
- **Complete Graph:** A graph in which every pair of distinct vertices is connected by an edge.
- **Proper Colouring of a Graph:** For a graph G, a *proper colouring* assigns colours to vertices so that adjacent vertices receive different colours.
- **k-Colourable Graph:** A graph that can be properly coloured using at most k colours such that no two adjacent vertices share the same colour.
- **Chromatic Number $\chi(G)$:** It is the minimum number of colours needed to get a proper colouring of a graph.

§5.2 Some Interesting Applications

1. **The Exam Scheduling Problem:** Your college has 6 courses: {Math, Physics, Chemistry, CS, English, History}. Some students take multiple courses. Can this be modelled as a graph colouring problem to schedule exams without conflicts?
2. **The Sudoku Connection:** Is solving a 4×4 Sudoku puzzle related to graph colouring?
3. **The Four Colour Theorem**
   To explore more on graph coloring, check out this video.