Deputy Coordinator Application

2025-26

Mathematics Club

Centre for Innovation
IIT Madras

Instructions

§1 General

- Provide descriptive answers for all questions. Explain your thought process well. Partial points will be awarded for partial solutions.
- You are free to use the internet and any other resources for solving the application. You are NOT allowed to collaborate with others. Please cite whatever reference(s) you use.
- The goal of the application is for you to learn as much as possible. Resources have been provided in the addendum for all sections. All the best and have fun!

§2 Application Format

- Your solutions can be written manually, in digital notes, documents, or in LaTeX. Upload all your scanned images, descriptive answers, and others in a single PDF file and submit the final document before the deadline. There is no weight to the mode of submission. **Spend more time on solving the app and not on beautifying it!**
- Keep your answers brief and elegant. Explain your thoughts concisely. Ensure not to exceed the word limit indicated next to the question.
- The PDF should be named `<Firstname>_<ROLLNUMBER(IN CAPS)>.pdf` (For example: `Anand_MC25H007.pdf`).

§3 Submission

- The final PDF is to be submitted in this Google form.
- The deadline for submission is **26th October, 2025, 23:59**.
- Join this WhatsApp group for further notifications and updates.

§4 Contact Us

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Contacts of all section-masters can be found in the addendum.

§0 General Questionnaire

1. Introduce yourself briefly, your interests, hobbies, anything interesting about you. *(under 50 words)*
2. Why do you want to be part of the Mathematics Club? What do you expect your DC tenure to look like? *(under 100 words)*
3. Have you been to any Mathematics Club event(s)? Briefly describe your experience in the event(s). *(under 150 words)*
4. Why do you like math? What topic do you like the most and which one do you hate the most in math? *(under 150 words)*
5. If you could propose your own mini-project for the club, which topics or domains would you like to explore, and why? *(under 100 words)*

§1 Infinite Possibilities

Ah, Probability, a topic we all claim to master... until we look deeper. Until now, you have probably only dealt with probability in the discrete sense with finite events. Think about throwing darts; there are infinitely many points it might land on. How can we talk about probability in this scenario?
We define a function $f_X(x)$ for a continuous Random Variable X, the Probability Density Function of X, which satisfies $\Pr(\, X \in A\,) = \int_A f_X(x)\,\mathrm{d}x$.

Problem 1.1. List down the necessary properties for a PDF (Probability Density Function).

Problem 1.2. Given the PDF $f_X(x) = \dfrac{e^{-(x-\mu)}}{\left(1+e^{-(x-\mu)}\right)^2}$, what is the CDF of the function? How would you represent the probability of finding numbers in a range using the CDF?

Problem 1.3. For any CDF, $F_X(x)$, I define the following integral

$$\int_0^\infty (1 - F_X(x) - F_X(-x))\,\mathrm{d}x$$

Evaluate the integral above for the PDF given in the previous problem. This integral evaluates to an interesting number for the distribution. Is this number special only for the given distribution, a specific set of distributions, or for all distributions? If it is true for all distributions, prove the same. If it is not true for all distributions, find all distributions that evaluate to this special number.

Problem 1.4. (*Bonus*) If I give you a Uniform distribution in the interval [0,1], how can you generate random numbers that follow a given probability distribution? Show why it follows the distribution.
(*Hint: The range of the CDF function is also* $[0, 1]$)

§2 Linear Algebra

We often think of math in terms of static numbers, but Linear Algebra is the language of transformation. It is used to describe change in a predictable and structured way. It is a truly beautiful field of math, showing that a simple set of rules can govern the complex operations that are used in countless applications, from the subtle colour shift in a digital photo to the massive calculations that power a search engine.

Problem 2.1. Hemanth has 4 vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4$ perpendicular to each another and forming the edges of a 4D unit hypercube. Its hyper-volume as you know is 1. Hemanth wants to transform these vectors to form a new 4D object.

1. **The First Transform:** Replace $\mathbf{e}_2$ with a new vector formed as $3\mathbf{e}_1 + 2\mathbf{e}_2$
2. **The Second Transform:** Replace $\mathbf{e}_4$ with the new vector $-\mathbf{e}_3$

After the second transform, Hemanth stops and sighs, "I have collapsed the object. The hypervolume is lost."

Problem 2.1.1. Explain, using the language of geometric transformations and linear algebra, *why* Hemanth is correct. What happened during the second weave that caused the hypervolume to become zero?
Problem 2.1.2. Hemanth wants to create a object with a final, non-zero hypervolume. He states that the first transform was perfect and must not be changed. However, he is willing to alter the second transform. What is the simplest possible modification to the second transform that will result in a final hypervolume of exactly -12? Justify your answer.

Problem 2.2. Infinite Candy Store: There is a magical candy store that operates under a curious rule. Whenever you visit the store and give them x chocolates and y toffees, they return to you

$$(\text{chocolates}, \text{toffees}) = (x + 3y,\, 8x + 3y).$$

You are allowed to visit the store only once per day. Each day, you hand over *all* the chocolates and toffees you have/received from the previous day to get new ones in return. You decided that you will eat everything only after n days when you will have exactly 2401 chocolates and 4802 toffees. Assume that the first time you visited the store (day 0), you had just one chocolate and t_0 toffees.

Problem 2.2.1. Find t_0 so that you'll eat 2401 chocolates and 4802 toffees some day in future.
Problem 2.2.2. Find $\min(n)$.
Problem 2.2.3.(*Bonus*) Prove that this is the only solution (t_0, n) possible.

§3 What do you call a Group of Mathematicians?

Refer to the addendum for resources.

Problem 3.1. Use the group axioms to prove that inverses are unique. (i.e every element has only one inverse)

Problem 3.2. Prove that for a finite group G with order $|G|$. The order of any subgroup of G divides $|G|$. Use the result to derive Fermat's little theorem.
Hint: Use subsets of $\mathbb{Z}_p^\times$ the multiplicative group modulo p for proving Fermat's little theorem.

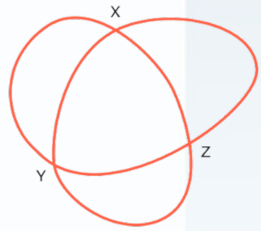
§4 To be or Knot to be?

Refer to the addendum for all necessary definitions.

Problem 4.1. What is the simplest possible, trivial, and non-trivial knot? Observe and comment on the crossing number of both knots.

Problem 4.2. Any 2 knots are said to be equivalent if one knot can be converted into another using a finite number of Reidemeister moves. Look at the two knots given below. Are both knots equivalent? If yes, can you sketch the Reidemeister moves to show it?

Problem 4.3. You observe the following shadow cast by a knot. The knot itself is not visible. For each intersection X, Y and Z, there is an equal chance of any one of the two segments being on top. What is the probability that the hidden knot is non-equivalent to the unknot?

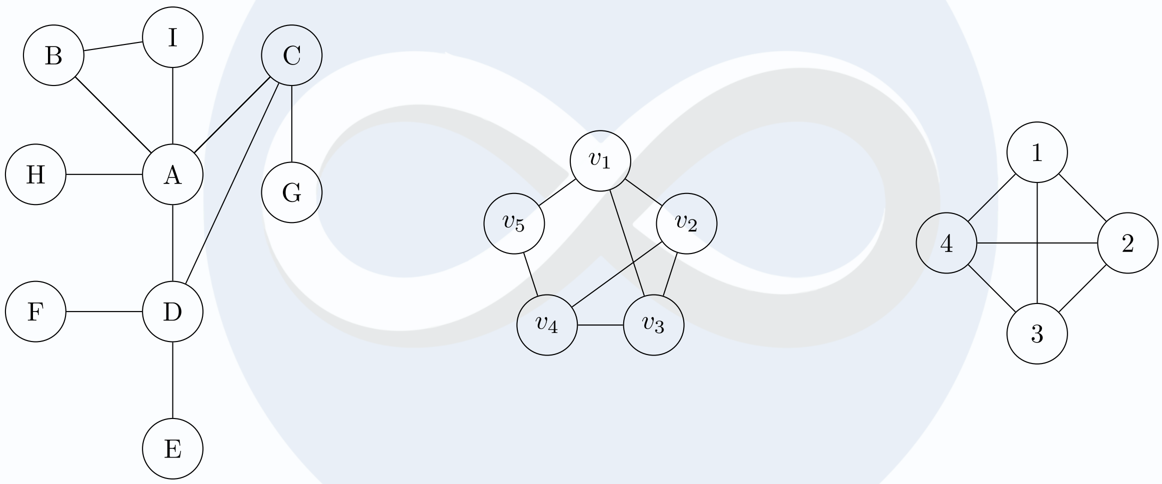


Problem 4.4. Back to Shoelaces (*Bonus*): Have you ever wondered why some people keep on stumbling due to loose shoelaces, and for some people, they seem to stay tied for eternity? What do you think the reason is? Turns out there is something hidden in the way people tie the *"knots"* of their shoelaces. Find out more and write the secret of the *"never unwinding **knots** of shoelaces"*.

§5 Help Me Colour: A Journey Through Graph Colouring

For basic definitions needed to solve these questions, go through the addendum.

Problem 5.1. Find the minimum number of colours required to give a proper colouring for each of the following graphs. Also, give one such proper colouring for each of the graphs given below using as few colours as possible.



Problem 5.2. Given a simple non-null graph, what will be the minimum number of colours that will always be required to properly colour it? You may use more colours but never fewer than this guaranteed minimum. Also give an example where the number of colours required to properly colour the graph is exactly equal to the specified minimum.

Problem 5.3. Peterson Graph: Peterson Graph is a very famous graph with some very special properties. Look up what the graph is and solve the below mentioned questions:

1. The Petersen graph is 3-colourable but not 2-colourable. Prove that it requires atleast 3 colours.
2. For graphs needing only 2 colours, what general observation can be made about their structure?

Problem 5.4. The Party Problem: In a party, some people are friends. We want to divide them into the minimum number of groups so that no two friends are in the same group. Can this problem be modeled along the lines of the problems we saw above, if yes, how? Model so for 6 people where friendships are:

$$(1,2),(2,3),(3,4),(4,5),(5,1),(1,4)$$

§6 Finite Structures

Problem 6.1. Consider the infinite sequence

$$7,\ 77,\ 777,\ 7777,\ \ldots$$

Prove that there exists at least one number in this sequence divisible by

$$393050634124102232869567034555427371542904833.$$

Problem 6.2. Let n, s, r, and p be positive integers with $n > s \cdot r \cdot p$. Show that any sequence of n real numbers contains at least one of the following:

- A strictly increasing subsequence of length greater than s,
- A strictly decreasing subsequence of length greater than r, or
- A constant subsequence of length greater than p.

(You are allowed to use the Erdős–Szekeres theorem. (~~Try proving Erdős–Szekeres using the pigeonhole principle.~~)

Problem 6.3. Construct explicit bijections $f : A \to B$ for the following cases:

(i) $A = [0, 1]$, $B = (0, 1)$.

(ii) A is the set of all strings using the Greek alphabet from α to ω, and B is the set of all Fibonacci numbers.

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===== END =====