



MathLab: Problem Set 1



MATHEMATICS CLUB IITM

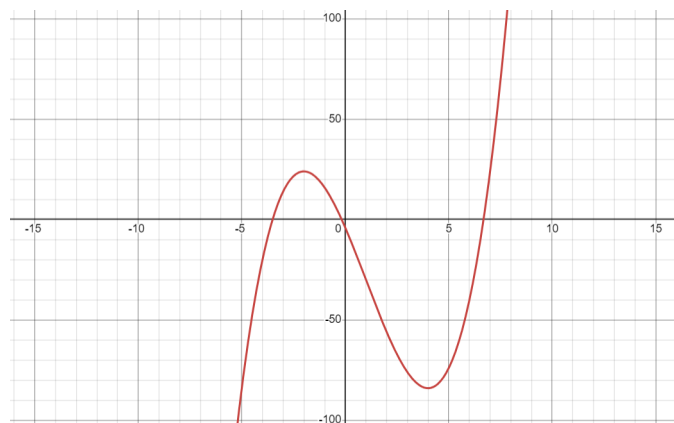
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Instructions

- This problem set consists of 4 sections and you need to solve **all 4 sections to complete this course**.
 - You are supposed to write your answers and upload it in the submission link given. You can submit a scanned copy of handwritten answers, submit a typed document or a handwritten document on a tablet. Anything works as long as it is legible and clear!
 - Try to answer all the questions in a clear and readable manner and mention all your assumptions/reasons explicitly.
 - It is fine even if you aren't able to solve the question completely after your best attempt. But **show us your working** or thought-process and the attempts that you have made in order **to clear the course**.
 - The deadline for submission is 19 June 2024, 10:30 a.m.
 - Feel free to reach out to us for doubts! Contact information of the problem-set creators:
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Problem 1: The Newton-Raphson Method



A plot of the function $f(x)$

- In the first session, we have covered the Newton-Raphson method which can be used to find roots of a function. Keeping that in mind answer the following questions.
- Our beloved coordinator Deena tries to solve the polynomial

$$f(x) = x^3 - 3x^2 - 24x - 4$$

with the initial guess $x_1 = 4$. Can he find the root of the polynomial?

- If yes, give the first two iterations i.e., x_2 and x_3
- If no, mention the reason

A plot of $f(x)$ has been given in the first page for your reference.

- Deena gives up on the initial value of $x_1 = 4$ and **chooses another** x_1 . He finds out that he cannot get a solution through Newton-Raphson method for the newly chosen value of x_1 . What is the value of the **newly chosen** x_1 ?

Problem 2: On Orders and Degrees

Find the Order and Degree of the following Differential Equations:

$$1. \left(\frac{d^3 y}{dx^3} \right)^2 + 2 \left(\frac{d^2 y}{dx^2} \right)^3 + 3 \left(\frac{dy}{dx} \right)^4 = 0$$

$$2. \left(\frac{d^4 y}{dx^4} \right)^{\frac{5}{7}} = 4 \left(\frac{d^3 y}{dx^3} \right)^2 - \left(\frac{dy}{dx} \right)$$

$$3. \sin \left(\frac{d^2 y}{dx^2} \right) + \left(\frac{dy}{dx} \right)^2 + y = 0$$

$$4. \left(\frac{d^2 y}{dx^2} \right)^3 + \left(\frac{dy}{dx} \right)^5 - e^{\frac{d^3 y}{dx^3}} = 0$$

Final Expected Answer: State the Order and Degree of the given Differential Equations and mention "Not defined" with an explanation whenever the Order or Degree is not defined.

Problem 3: Classification Conundrums

Classify the following Differential Equations based on Linearity and Homogeneity.

$$1. \frac{d^2 u}{dx^2} + \frac{u}{1+x^2} = 0$$

$$2. y'' + 3y' - 4y = \sin(x)$$

$$3. y'' + e^y y' + \cos(y) = \cos(x)$$

$$4. y'' + \sin(y)y' + y^3 = 0$$

Final Expected Answer: State if the given Differential Equations are i) Linear or Non-linear and ii) Homogenous or Non-homogenous.

Problem 4: Pradyumnan's Interests



Pradyumnan borrowed Rs. 6,40,000 to buy a car. The lender charges interest at an annual rate of 10%. Assuming that interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate k , determine the payment rate k that is required to pay off the loan in 3 years. Also determine how much interest is paid during the 3-year period.

Note that by “continuously” we mean every passing instant of time. Every nanosecond counts!

Set up equations and show your working clearly while solving this question.